Package 'elastes'

July 3, 2022

Type Package

Title Elastic Full Procrustes Means for Sparse and Irregular Planar Curves

Version 0.1.6

Description Provides functions for the computation of functional elastic shape means over sets of open planar curves. The package is particularly suitable for settings where these curves are only sparsely and irregularly observed. It uses a novel approach for elastic shape mean estimation, where planar curves are treated as complex functions and a full Procrustes mean is estimated from the corresponding smoothed Hermitian covariance surface. This is combined with the methods for elastic mean estimation proposed in Steyer, Stöcker, Greven (2022) <doi:10.1111/biom.13706>. See Stöcker et. al. (2022) <arXiv:2203.10522> for details.

License GPL (>= 3)

Encoding UTF-8

Language en-US

Imports elasdics, utils, graphics, stats, splines, mgcv, sparseFLMM, orthogonalsplinebasis

Suggests knitr, covr, testthat (>= 3.0.0), rmarkdown, shapes

RoxygenNote 7.2.0

Config/testthat/edition 3

VignetteBuilder knitr

URL https://mpff.github.io/elastes/

NeedsCompilation no

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Repository CRAN

Date/Publication 2022-07-03 18:20:05 UTC

R topics documented:

`compute_elastic_shape_mean`

Compute an elastic full Procrustes mean for a collection of curves

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Description

Computes an elastic full Procrustes mean for curves stored in `data_curves`. Constructor function for class `elastic_shape_mean`.

Usage

```
compute_elastic_shape_mean(
  data_curves,
  knots = seq(0, 1, len = 13),
  type = c("smooth", "polygon"),
  penalty = 2,
  var_type = c("smooth", "constant", "zero"),
  pfit_method = c("smooth", "polygon"),
  smooth_warp = function(i) 0,
  eps = 0.05,
  max_iter = 50,
  verbose = FALSE,
  cluster = NULL
)
```

Arguments

`data_curves`	list of `data.frames` with observed points in each row. Each variable is one coordinate direction. If there is a variable `t`, it is treated as the time parametrization, not as an additional coordinate.
`knots`	set of knots for the mean spline curve

type	if "smooth" linear srv-splines are used which results in a differentiable mean curve if "polygon" the mean will be piecewise linear.
penalty	the penalty to use in the covariance smoothing step. use '-1' for no penalty.
var_type	(experimental) assume "smooth", "constant" or "zero" measurement-error variance along t
pfit_method	(experimental) "smooth" or "polygon"
smooth_warp	(experimental) controls the weighting of original and smoothed observations over the iterations, if pfit_method == "smooth".
eps	the algorithm stops if L2 norm of coefficients changes by less than eps
max_iter	maximal number of iterations
verbose	print iterations
cluster	(experimental) use the parallel package for faster computation

Value

an object of class elastic_shape_mean, which is a list with entries

type	"smooth" if mean was modeled using linear srv-splines, "polygon" if constant srv-splines
coefs	spline coefficients
knots	spline knots
variance	sample elastic shape variance
data_curves	list of data.frames with observed points in each row. First variable t gives the initial parametrization, second variable t_optim the optimal parametrization when the curve is aligned to the mean. Has the attributes 'rotation', 'scaling', 'translation' and 'dist_to_mean'. Use get_procrustes_fit to get the elastic full Procrustes fit.
fit	see fit_mean

Examples

```
curve <- function(t){
  rbind(t*cos(13*t), t*sin(13*t))
}
set.seed(18)
data_curves <- lapply(1:4, function(i){
  m <- sample(10:15, 1)
  delta <- abs(rnorm(m, mean = 1, sd = 0.05))
  t <- cumsum(delta)/sum(delta)
  data.frame(t(curve(t)) + 0.07*t*matrix(cumsum(rnorm(2*length(delta))),
             ncol = 2))
})

#randomly rotate and scale curves
rand_scale <- function(curve){ ( 0.5 + runif(1) ) * curve }
rand_rotate <- function(curve){
  names <- colnames(curve)
```

```
  theta <- 2*pi*runif(1)
  mat <- matrix(c(cos(theta), sin(theta), -sin(theta), cos(theta)), nrow = 2, ncol = 2)
  curve.rot <- as.matrix(curve) %*% t(mat)
  curve.rot <- as.data.frame(curve.rot)
  colnames(curve.rot) <- names
  return(curve.rot)
}
data_curves <- lapply(data_curves, rand_scale)
data_curves <- lapply(data_curves, rand_rotate)

#compute smooth procrustes mean with 2 order penalty
knots <- seq(0,1, length = 11)
elastic_shape_mean <- compute_elastic_shape_mean(
    data_curves,
    knots = knots,
    type = "smooth",
    penalty = 2
    )
plot(elastic_shape_mean)
```

`elastes` *elastes: Elastic Full Procrustes Means for Sparse and Irregular Planar Curves*

Description

Provides functions for the computation of functional elastic shape means over sets of open planar curves. The package is particularly suitable for settings where these curves are only sparsely and irregularly observed. It uses a novel approach for elastic shape mean estimation, where planar curves are treated as complex functions and a full Procrustes mean is estimated from the corresponding smoothed hermitian covariance surface, which is combined with the methods for elastic mean estimation proposed in Steyer, Stöcker, Greven (2022). See Stöcker et. al. (2022) for details on the method.

Details

Compute a mean for a set of observed curves: `compute_elastic_shape_mean`

`fit_alignment_proc2d` *Optimal rotation and scaling alignment to a smooth curve*

Description

Finds optimal rotation and scaling alignment for a discrete open srv curve to a smooth curve

Usage

```
fit_alignment_proc2d(
  q,
  type,
  knots,
  var_type,
  coefs.compl,
  method,
  cov_fit,
  pca,
  L
)
```

Arguments

q	complex srv curve with parametrization, needs to be vectorized. The result of a call to get_model_data_complex
type	spline degree
knots	basis knots
var_type	either "smooth" or "constant" measurement error in cov_fit object
coefs.compl	complex coefficients of smooth curve
method	temp
cov_fit	temp
pca	temp
L	temp

Value

optimal rotation G and scaling b

---

fit_mean *Mean estimation for open planar curves.*

Description

Fits an elastic full Procrustes mean for open, planar curves. Is usually called from compute_elastic_shape_mean.

Usage

```
fit_mean(
  srv_data_curves,
  knots,
  penalty,
  var_type,
```

```
  pfit_method,
  max_iter,
  type,
  eps,
  cluster,
  verbose,
  smooth_warp
)
```

Arguments

srv_data_curves	list of data.frames with srv vectors in each row.curves
knots	set of knots for the mean spline curve
penalty	the penalty to use in the covariance smoothing step. use '-1' for no penalty.
var_type	(experimental) assume "smooth", "constant" or "zero" measurement-error variance along t
pfit_method	(experimental) "smooth" or "polygon"
max_iter	maximal number of iterations
type	if "smooth" linear srv-splines are used which results in a differentiable mean curve if "polygon" the mean will be piecewise linear.
eps	the algorithm stops if L2 norm of coefficients changes less
cluster	a cluster object for use in the bam call
verbose	print iterations
smooth_warp	(experimental) controls the weighting of original and smoothed observations over the iterations, if pfit_method == "smooth".

Value

a list with entries

type	"smooth" or "polygon"
coefs	coefs srv spline coefficients of the estimated mean
knots	spline knots
penalty	penalty used in the covariance estimation
distances	distances to mean
fit	a list containing t_optimsoptimal parametrizations G_optimsoptimal rotations b_optimsoptimal scalings n_optimsoptimal re-normalization n_iternumber of iterations until convergence gram the mean basis Gram matrix, cov_fit the covariance smoothing objects in the final iteration, cov_pca cov coef matrix pca object in the final iteration and pfit_coefs the mean basis coefs of smoothed pfits in the final iteration

get_center *Calculate the center of a curve*

Description

Calculate the center of a curve

Usage

```
get_center(curve)
```

Arguments

curve	a data.frame with observed points in each row. Each variable is one coordinate direction. If there is a variable t, t_optim or id, it is treated as the time parametrization, not as an additional coordinate.

Value

The average of observed points in curve.

get_distance *Distance to a smooth curve*

Description

Finds the distance of a discrete open srv curve to a smooth curve

Usage

```
get_distance(srv_curve, s, q, eps = 10 * .Machine$double.eps)
```

Arguments

srv_curve	srv transformation of the smooth curve, needs to be vectorized
s	time points for q, first has to be 0, last has to be 1
q	square root velocity vectors, one less than time points in s
eps	convergence tolerance

Value

distance between srv_curve and q

get_evals *Evaluate a curve on a grid*

Description

Evaluate a curve on a grid

Usage

```
get_evals(curve, t_grid = NULL, ...)

## S3 method for class 'data.frame'
get_evals(curve, t_grid = NULL, ...)

## S3 method for class 'elastic_shape_mean'
get_evals(curve, t_grid = NULL, centering = TRUE, srv = FALSE, ...)
```

Arguments

curve	a one parameter function which is to be evaluated on a grid
t_grid	the curve is evaluated at the values in t_grid, first value needs to be 0, last value needs to be 1. If t_grid = NULL, a default regular grid with grid length 0.01 is chosen
...	other arguments
centering	TRUE if curves shall be centered
srv	TRUE if SRV curve shall be evaluated

Value

a data.frame with evaluations of the curve at the values in t_grid in its rows.

See Also

See get_evals for the original code.

Examples

```
curve <- function(t){c(t*sin(10*t), t*cos(10*t))}
plot(get_evals(curve), type = "b")
```

get_optimal_t *Finds optimal alignment for discrete open curves*

Description

Finds optimal aligned time points for srv curve q to srv curve p using coordinate wise optimization.

Usage

```
get_optimal_t(srv_procrustes_curves, coefs, t_optims, type, knots, eps, i)
```

Arguments

srv_procrustes_curves	scaling and rotation aligned srv curves
coefs	mean coefficients
t_optims	current optimal parametrization
type	"smooth" or "polygon"
knots	mean basis knots
eps	convergence tolerance
i	current iteration

Value

optimal time points for srv_data_curves, without first value 0 and last value 1 optimal time points have the distance of the observation to the srv_curve as an attribute

get_polygon_length *Calculate the polygon length of a curve*

Description

Calculate the polygon length of a curve

Usage

```
get_polygon_length(curve)
```

Arguments

curve	a data.frame with observed points in each row. Each variable is one coordinate direction. If there is a variable t, t_optim or id, it is treated as the time parametrization, not as an additional coordinate.

Value

The length of curve, treating it as a polygon.

get_procrustes_fit *Get Procrustes data curve from mean object.*

Description

Compute the Procrustes aligned data curve...

Usage

```
get_procrustes_fit(data_curve)
```

Arguments

data_curve	A data.frame in an elastic_shape_mean object.

Value

Aligned data_curve as a data.frame.

get_Procrustes_fit_from_param *Helper functions for calculating Procrustes data curve from rotation, scaling and translation parameters.*

Description

Compute the Procrustes fit given optimal rotation, scaling and translation.

Usage

```
get_procrustes_fit_from_param(
  data_curve,
  rot,
  scale,
  plength,
  trans,
  norm_factor
)
```

Arguments

data_curve	A data.frame with observed points on a curve. Each row is one point, each variable one coordinate direction. If there is a variable t, it is treated as the time parametrization, not as an additional coordinate.
rot	The rotation (in radian).
scale	The scaling.
plength	The polygon length of the original curve.
trans	The translation.
norm_factor	The normalization factor from the smooth curve estimate.

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plot.elastic_shape_mean

Plot method for planar elastic Procrustes mean curves

Description

Plots objects of class elastic_shape_mean.

Usage

```
## S3 method for class 'elastic_shape_mean'
plot(x, srv = FALSE, centering = TRUE, asp = 1, col = "red", ...)
```

Arguments

x	object of class elastic_shaped_mean, usually a result of a call to compute_elastic_shape_mean
srv	TRUE if the SRV curve should be plotted
centering	TRUE if mean and pfits should be centered
asp	numeric, giving the aspect ratio of the two coordinates, see plot.window for details.
col	color of the mean curve.
...	further plotting parameters.

Value

No return value, called for side effects.

See Also

For examples see documentation of compute_elastic_shape_mean. See plot.elastic_mean for the original code.

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