

Package ‘RankAggSIgFUR’

May 21, 2022

Title Polynomially Bounded Rank Aggregation under Kemeny's Axiomatic Approach

Version 0.1.0

Description Polynomially bounded algorithms to aggregate complete rankings under Kemeny's axiomatic framework. 'RankAggSIgFUR' (pronounced as rank-agg-cipher) contains two heuristics algorithms: FUR and SIgFUR. For details, please see Badal and Das (2018) <[doi:10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)>.

License GPL (>= 3)

Encoding UTF-8

RoxygenNote 7.2.0

Depends R (>= 2.10)

LazyData true

Imports Rfast, combinat

Suggests testthat (>= 3.0.0)

Config/testthat/edition 3

NeedsCompilation yes

URL <https://github.com/prakashvs613/RankAggSIgFUR>

BugReports <https://github.com/prakashvs613/RankAggSIgFUR/issues>

Author Hannah Parker [aut],
Rakhi Singh [cre, aut] (<<https://orcid.org/0000-0003-3469-295X>>),
Prakash Singh Badal [aut] (<<https://orcid.org/0000-0003-4097-8444>>)

Maintainer Rakhi Singh <agrakhi@gmail.com>

Repository CRAN

Date/Publication 2022-05-21 15:30:02 UTC

R topics documented:

data100x15	2
data240x4	3

data400x15	4
data50x15	5
fur	6
mean_seed	8
mod_kemeny	9
rap_greedy_alg	10
seed_based_iteration	11
sigfur	12
subit_convergence	14

Index	16
--------------	-----------

data100x15	<i>Simulated 100 × 15 Data</i>
------------	--------------------------------

Description

Data of 100 objects and 15 attributes, in which the first column contains the object names and each subsequent column is a complete ranking of the 100 objects. The included 50×15 and 400×15 datasets were generated from this dataset (see [data50x15](#) and [data400x15](#)).

Usage

```
data(data100x15)
```

Format

A data frame with 100 rows and 16 columns:

- Object** object name
- Ranking 1** ranking on the first attribute
- Ranking 2** ranking on the second attribute
- Ranking 3** ranking on the third attribute
- Ranking 4** ranking on the fourth attribute
- Ranking 5** ranking on the fifth attribute
- Ranking 6** ranking on the sixth attribute
- Ranking 7** ranking on the seventh attribute
- Ranking 8** ranking on the eighth attribute
- Ranking 9** ranking on the ninth attribute
- Ranking 10** ranking on the tenth attribute
- Ranking 11** ranking on the eleventh attribute
- Ranking 12** ranking on the twelfth attribute
- Ranking 13** ranking on the thirteenth attribute
- Ranking 14** ranking on the fourteenth attribute
- Ranking 15** ranking on the fifteenth attribute

Source

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

Examples

```
data(data100x15)
input_rkgs <- t(as.matrix(data100x15[, -1]))
obj_names <- data100x15[,1]

# Determine the mean seed ranking
mean_seed(input_rkgs)
```

data240x4

PrefLib 240 × 4 Data

Description

Data of 240 cities across the globe ranked on four criteria from the ED-00015-001.soc dataset in the PrefLib repository. The first column contains the object names and each subsequent column is a complete ranking of the 240 objects with no ties) .

Usage

```
data(data240x4)
```

Format

A data frame with 240 rows and 5 columns:

Object object name

Ranking 1 ranking on the first criterion

Ranking 2 ranking on the second criterion

Ranking 3 ranking on the third criterion

Ranking 4 ranking on the fourth criterion

Source

<https://www.preflib.org/>

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

Mattei, N., & Walsh, T. (2013, November). Preflib: A library for preferences <https://www.preflib.org/>. In *International conference on algorithmic decision theory* (pp. 259-270). Springer, Berlin, Heidelberg.

Examples

```
data(data240x4)
input_rkgs <- t(as.matrix(data240x4[, -1]))
obj_names <- data240x4[,1]

# Determine the mean seed ranking
mean_seed(input_rkgs)
```

data400x15

Simulated 400 × 15 Data

Description

Data of 400 objects and 15 attributes in which the first column contains the object names and each subsequent column is a complete ranking of the 400 objects. This data set is generated from the 100 × 15 dataset (see [data50x15](#)) by adding 100 to the ranks of the objects numbered 1 through 100 to get the ranks of objects numbered 101 through 200. Similarly, by adding 200 to obtain ranking 201 through 300, and by adding 300 to obtain ranking 301 through 400.

Usage

```
data(data400x15)
```

Format

A data frame with 400 rows and 16 columns:

Objects object name

Ranking 1 ranking on the first attribute

Ranking 2 ranking on the second attribute

Ranking 3 ranking on the third attribute

Ranking 4 ranking on the fourth attribute

Ranking 5 ranking on the fifth attribute

Ranking 6 ranking on the sixth attribute

Ranking 7 ranking on the seventh attribute

Ranking 8 ranking on the eighth attribute

Ranking 9 ranking on the ninth attribute

Ranking 10 ranking on the tenth attribute

Ranking 11 ranking on the eleventh attribute

Ranking 12 ranking on the twelfth attribute

Ranking 13 ranking on the thirteenth attribute

Ranking 14 ranking on the fourteenth attribute

Ranking 15 ranking on the fifteenth attribute

Source

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

Examples

```
data(data400x15)
input_rkgs <- t(as.matrix(data400x15[, -1]))
obj_names <- data400x15[,1]

# Determine the mean seed ranking
mean_seed(input_rkgs)
```

data50x15

Simulated 50 × 15 Data

Description

Data of 50 objects and 15 attributes, which were randomly generated from the 100×15 simulated dataset (see [data100x15](#)). The first column contains the object names and each subsequent column is a complete ranking of the 50 objects.

Usage

```
data(data50x15)
```

Format

A data frame with 50 rows and 16 columns:

Object object name
Ranking 1 ranking on the first attribute
Ranking 2 ranking on the second attribute
Ranking 3 ranking on the third attribute
Ranking 4 ranking on the fourth attribute
Ranking 5 ranking on the fifth attribute
Ranking 6 ranking on the sixth attribute
Ranking 7 ranking on the seventh attribute
Ranking 8 ranking on the eighth attribute
Ranking 9 ranking on the ninth attribute
Ranking 10 ranking on the tenth attribute
Ranking 11 ranking on the eleventh attribute
Ranking 12 ranking on the twelfth attribute
Ranking 13 ranking on the thirteenth attribute
Ranking 14 ranking on the fourteenth attribute
Ranking 15 ranking on the fifteenth attribute

Source

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

Examples

```
data(data50x15)
input_rkgs <- t(as.matrix(data50x15[, -1]))
obj_names <- data50x15[,1]

# Determine the mean seed ranking
mean_seed(input_rkgs)
```

fur

FUR

Description

FUR is a heuristic algorithm to obtain a consensus ranking. It contains three branches – Fixed, Update, and Range – that use *Subiterative Convergence* and *Greedy Algorithm* iteratively. See ‘Details’ for more information on each branch.

Usage

```
fur(input_rkgs, subit_len_list, search_radius, seed_rkg = c())
```

Arguments

input_rkgs	a n by k matrix of k rankings of n objects, where each column is a complete ranking.
subit_len_list	a vector containing positive integer(s) for the subiteration lengths to <i>Subiterative Convergence</i> . Recommended values are between 2 and 8. Smaller subiteration lengths result in shorter run-time.
search_radius	a positive integer for the maximum change in the rank of each object in the <i>Greedy Algorithm</i> . The default value of 0 considers all possible rank changes for each object. It is recommended to use a search radius of less than or equal to $\min(30, \lfloor n/2 \rfloor)$.
seed_rkg	a vector of length n with an initial ranking to begin FUR. If the default value of an empty vector is used, then the mean seed ranking is adopted as the initial ranking to FUR.

Details

The Fixed branch applies *Subiterative Convergence* using one subiteration length from `subit_len_list` at a time.

The Update branch executes *Subiterative Convergence* using the first subiteration length in `subit_len_list`, and then uses its output in the next call to *Subiterative Convergence* with the next subiteration length in the list. This process repeats until `subit_len_list` is exhausted.

The Range branch calls *Subiterative Convergence* on all subiteration lengths in `subit_len_list` and only retains the best ranking among these separate calls.

The output from the *Subiterative Convergence* calls are fed into the *Greedy Algorithm* as its seed ranking, and the FUR algorithm is terminated when the input to the *Greedy Algorithm* converges to the output and all branches have been executed at least once.

Value

A list containing the consensus ranking (expressed as ordering), total Kemeny distance, and average tau correlation coefficient corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

See Also

[mean_seed](#), [subit_convergence](#), [rap_greedy_alg](#), [sigfur](#)

Examples

```
## One subiteration length
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
subit_len_list <- 2
search_radius <- 1
fur(input_rkgs, subit_len_list, search_radius) # Determined the consensus ranking, total Kemeny
# distance, and average tau correlation coefficient

## Multiple subiteration lengths
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
subit_len_list <- c(2,3)
search_radius <- 1
fur(input_rkgs, subit_len_list, search_radius)

## Included dataset of 15 input rankings of 50 objects
data(data50x15)
input_rkgs <- as.matrix(data50x15[, -1])
subit_len_list <- c(2, 3)
search_radius <- 1
fur(input_rkgs, subit_len_list, search_radius)
```

`mean_seed`*Mean Seed Ranking*

Description

Determine the *mean seed ranking* of the given input rankings. The average rank of an object is the sum of its various rankings from each input ranking divided by the total number of rankings. The mean seed ranking is formed by ranking the objects based on their average ranks, and ties are broken by ranking the first tied object with a higher rank.

Usage

```
mean_seed(input_rkgs)
```

Arguments

`input_rkgs` a k by n matrix of k rankings of n objects, where each row is a complete ranking. Note that this is a transpose of matrix used for functions like `fur`, `sigfur`, `rap_greedy_alg`, and `subit_convergence`.

Value

A vector containing the mean seed ranking of the input rankings.

See Also

[rank](#), [subit_convergence](#), [fur](#), [sigfur](#)

Examples

```
## Four input rankings of five objects
input_rkgs <- matrix(c(3, 2, 5, 1, 2, 3, 1, 2, 5, 1, 3, 4, 4, 5, 4, 5, 1, 4, 2, 3), ncol = 5)
mean_seed(input_rkgs) # Found the mean seed ranking

## Included dataset of 15 input rankings of 50 objects
data(data50x15)
input_rkgs <- t(as.matrix(data50x15[, -1]))
mean_seed(input_rkgs)
```

`mod_kemeny`*Modified Kemeny Rank Aggregation*

Description

Modified Kemeny algorithm determines the consensus ranking of n objects using the set of all possible rankings compared to the input rankings. The algorithm is based on Kemeny's axiomatic approach of minimizing the total Kemeny distance from the input rankings. In case of multiple rankings with minimum total Kemeny distance, the consensus ranking is determined using two additional criteria. See 'Details' for additional criteria. The method involves $n!$ comparisons. Hence, it works best on a set of rankings with a small number of objects.

Usage

```
mod_kemeny(input_rkgs, universe_rkgs, obj_pairs)
```

Arguments

<code>input_rkgs</code>	a k by n matrix of k rankings of n objects, where each row is a complete ranking. Note that this is a transpose of matrix used for functions like <code>fur</code> , <code>sigfur</code> , <code>rap_greedy_alg</code> , and <code>subit_convergence</code> .
<code>universe_rkgs</code>	a matrix containing all possible permutations of ranking n objects. Each row in this matrix represents one permuted ranking.
<code>obj_pairs</code>	a 2 by n choose 2 matrix of all combinations of object pairs of n objects, where each column contains a pair of object indices.

Details

Under Kemeny's axiomatic approach, rankings with minimum total Kemeny distance are considered equally optimal. Modified Kemeny attempts to break the tie among such rankings by imposing two additional criteria on the basis of minimizing (a) the maximum and (b) the variance of individual Kemeny distances, applied sequentially.

Value

A list containing the consensus ranking (expressed as ordering), total Kemeny distance, and average tau correlation coefficient corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

Examples

```
## Consensus ranking from four rankings of five objects
n <- 5
input_rkgs <- matrix(c(3, 2, 5, 1, 2, 3, 1, 2, 5, 1, 3, 4, 4, 5, 4, 5, 1, 4, 2, 3), ncol = n)
uni_rkgs <- matrix(unlist(combinat::permn(c(1:n))), byrow = TRUE, ncol = n)
obj_pairs <- combinat::combn(1:n, 2, simplify=TRUE)
mod_kemeny(input_rkgs, uni_rkgs, obj_pairs) # Computed consensus ranking, total Kemeny distance,
# and average tau correlation coefficient
```

rap_greedy_alg

Greedy Algorithm for Rank Aggregation

Description

Greedy Algorithm is a heuristic method that hunts for improved rankings by moving one object at a time (up or down). In case an object's movement results in an improved ranking, the next object is moved with respect to this improved ranking. The process is repeated until all objects are considered once.

Usage

```
rap_greedy_alg(seed_rkg, input_rkgs, search_radius = 0)
```

Arguments

seed_rkg	an initial ranking to begin the algorithm. The algorithm is often used in conjunction with <i>Subiterative Convergence</i> .
input_rkgs	a n by k matrix of k rankings of n objects, where each column is a complete ranking.
search_radius	a positive integer for the maximum change in the rank of each object. The default value of 0 considers all possible rank changes for each object. Recommended value of search radius is less than or equal to $\min(30, \lfloor n/2 \rfloor)$.

Value

A list containing the consensus ranking (expressed as ordering), total Kemeny distance, and average tau correlation coefficient corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

See Also

[subit_convergence](#), [fur](#), [sigfur](#)

Examples

```
## Four input rankings of five objects
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
mean_seed_rkg <- mean_seed(t(input_rkgs))
rap_greedy_alg(mean_seed_rkg, input_rkgs, search_radius = 0) # Determined the consensus ranking,
# total Kemeny distance, and average
# tau correlation coefficient

## Included dataset of 15 input rankings of 50 objects
data(data50x15)
input_rkgs <- as.matrix(data50x15[, -1])
mean_seed_rkg <- mean_seed(t(input_rkgs)) # Use the mean seed ranking as the seed ranking
rap_greedy_alg(mean_seed_rkg, input_rkgs, search_radius = 1)
```

seed_based_iteration *Seed-Based Iteration*

Description

Seed-Based Iteration is a heuristic-based seed generation used in *SIGFUR* to iteratively perturb the ranking to improve the consensus ranking.

Usage

```
seed_based_iteration(eta, omega, input_rkgs)
```

Arguments

eta	a subiteration length for intermittent <i>Subiterative Convergence</i> . The recommended values are between 2 and 8. Smaller subiteration lengths result in shorter run-time.
omega	a positive integer for the number of repetitions of perturbing the seed ranking. An omega value of 1 corresponds to a single application of <i>Subiterative Convergence</i> .
input_rkgs	a k by n matrix of k rankings of n objects, where each row is a complete ranking. Note that this is a transpose of matrix used for functions like <i>fur</i> , <i>sigfur</i> , <i>rap_greedy_alg</i> , and <i>subit_convergence</i> .

Value

A list containing the consensus ranking (expressed as ordering) and total Kemeny distance corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

See Also

[sigfur](#), [subit_convergence](#), [mean_seed](#)

Examples

```
## Four input rankings of five objects
eta <- 2
omega <- 10
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
seed_based_iteration(eta, omega, t(input_rkgs)) # Determined seed-based iterations

## Included dataset of 15 input rankings of 50 objects
eta <- 3
omega <- 5
data(data50x15)
input_rkgs <- as.matrix(data50x15[, -1])
seed_based_iteration(eta, omega, t(input_rkgs)) # Determined seed-based iterations
```

sigfur

SigFUR

Description

SigFUR applies *Seed-Based Iteration*, *Greedy Algorithm*, and *FUR* in sequence for each element of `subit_len_list_sbi`. The *mean seed ranking* is used as the input to *Seed-Based Iteration*. The best of all output rankings from *FUR* is considered as the consensus ranking.

Usage

```
sigfur(
  input_rkgs,
  subit_len_list_sbi,
  omega_sbi,
  subit_len_list_fur,
  search_radius
)
```

Arguments

`input_rkgs` a n by k matrix of k rankings of n objects, where each column is a complete ranking.

`subit_len_list_sbi` a vector containing positive integer(s) for the subiteration lengths to *Seed-Based Iteration*. Recommended values are between 2 and 8. Smaller subiteration lengths result in shorter run-time.

- `omega_sbi` a positive integer for the number of repetitions of perturbing the seed ranking in *Seed-Based Iteration*. An `omega_sbi` value of 1 corresponds to a single application of *Subiterative Convergence*.
- `subit_len_list_fur` a vector containing positive integer(s) for the subiteration lengths to *FUR*.
- `search_radius` a positive integer for the maximum change in the rank of each object in the *Greedy Algorithm* and *FUR*. The default value of 0 considers all possible rank changes for each object. It is recommended to use a search radius of less than or equal to $\min(30, \lfloor n/2 \rfloor)$.

Value

A list containing the consensus ranking (expressed as ordering), total Kemeny distance, and average tau correlation coefficient corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

See Also

[seed_based_iteration](#), [rap_greedy_alg](#), [fur](#), [mean_seed](#)

Examples

```
## Four input rankings of five objects
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
subit_len_list_sbi <- c(2:3)
omega_sbi <- 10
subit_len_list_fur <- c(2:3)
search_radius <- 1
sigfur(input_rkgs, subit_len_list_sbi, omega_sbi, subit_len_list_fur, search_radius)
# Determined the consensus ranking, total Kemeny distance, and average tau correlation coefficient

## Included dataset of 15 input rankings of 50 objects
data(data50x15)
input_rkgs <- as.matrix(data50x15[, -1])
subit_len_list_sbi <- c(3)
omega_sbi <- 5
subit_len_list_fur <- c(2:3)
search_radius <- 1
sigfur(input_rkgs, subit_len_list_sbi, omega_sbi, subit_len_list_fur, search_radius)
```

subit_convergence *Subiterative Convergence*

Description

Subiterative Convergence finds the consensus ranking by iteratively applying the *Modified Kemeny* algorithm on smaller number of objects, η . Starting with a given seed ranking, the consensus ranking is obtained when the algorithm converges.

Usage

```
subit_convergence(eta, seed_rkg, input_rkgs, universe_rkgs = c())
```

Arguments

eta	a subiteration length of number of objects to consider in the smaller subset. Recommended eta values are between 2 and 8. Smaller eta values result in shorter run-time.
seed_rkg	an initial ranking to start the algorithm. An ideal seed ranking for <i>Subiterative Convergence</i> is the <i>mean seed ranking</i> of input rankings.
input_rkgs	a n by k matrix of k rankings of n objects, where each column is a complete ranking.
universe_rkgs	a matrix containing all possible permutations of ranking n objects. Each column in this matrix represents one permuted ranking.

Value

A list containing the consensus ranking (expressed as ordering), total Kemeny distance, and average tau correlation coefficient corresponding to the consensus ranking.

References

Badal, P. S., & Das, A. (2018). Efficient algorithms using subiterative convergence for Kemeny ranking problem. *Computers & Operations Research*, 98, 198-210. doi: [10.1016/j.cor.2018.06.007](https://doi.org/10.1016/j.cor.2018.06.007)

See Also

[mod_kemeny](#), [fur](#), [sigfur](#), [mean_seed](#)

Examples

```
## Four input rankings of five objects
eta <- 3
seed_rkg <- c(1, 2, 3, 4, 5)
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
subit_convergence(eta, seed_rkg, input_rkgs) # Determined the consensus ranking, total Kemeny
```

```
# distance, and average tau correlation coefficient

## Example with eta=1
eta <- 1
seed_rkg <- c(1, 2, 3, 4, 5)
input_rkgs <- matrix(c(3, 2, 5, 4, 1, 2, 3, 1, 5, 4, 5, 1, 3, 4, 2, 1, 2, 4, 5, 3),
  byrow = FALSE, ncol = 4)
subit_convergence(eta, seed_rkg, input_rkgs) # Shows a warning and returns seed ranking

## Included dataset of 15 input rankings of 50 objects
data(data50x15)
input_rkgs <- as.matrix(data50x15[, -1])
mean_seed_rkg <- mean_seed(t(input_rkgs)) # Use the mean seed ranking as the seed ranking
eta <- 2
subit_convergence(eta, seed_rkg = mean_seed_rkg, input_rkgs)
```

Index

* datasets

data100x15, [2](#)

data240x4, [3](#)

data400x15, [4](#)

data50x15, [5](#)

data100x15, [2](#), [5](#)

data240x4, [3](#)

data400x15, [2](#), [4](#)

data50x15, [2](#), [4](#), [5](#)

fur, [6](#), [8](#), [10](#), [13](#), [14](#)

mean_seed, [7](#), [8](#), [12–14](#)

mod_kemeny, [9](#), [14](#)

rank, [8](#)

rap_greedy_alg, [7](#), [10](#), [13](#)

seed_based_iteration, [11](#), [13](#)

sigfur, [7](#), [8](#), [10](#), [12](#), [12](#), [14](#)

subit_convergence, [7](#), [8](#), [10](#), [12](#), [14](#)